# Chapter 10

# Sequences and Series

$$a_n = a_1 + (n-1)d$$

$$a_n = a_1 \bullet r^{n-1}$$

$$S_n = n\left(\frac{a_1 + a_n}{2}\right)$$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S = \frac{a_1}{1-r}$$

# Section 10-1
# Sequences as Functions

**Sequence:**

**Term:**

**Finite sequence:**

**Infinite sequence**:

**Arithmetic sequence:**

**Common difference:**

**Find the next 4 terms of the arithmetic sequence and state the common difference.**

| | |
|---|---|
| **1. 18, 11, 4, ....** | **2. 7, 13, 19, ...** |

**Geometric sequence:**

**Common ratio:**

**Find the next 3 terms of the geometric sequence and state the common ratio.**

| | |
|---|---|
| **1. 32, 8, 2, ....** | **2. 8, 20, 50, 125, ...** |

**State whether the following sequences are arithmetic or geometric. Why?**

| | |
|---|---|
| **1. -8, -2, 4, 10, ...** | **2. 6, 20, 50, 125, ...** |

# Section 10-2
# Arithmetic Sequence and Series

## Part 1: Arithmetic Sequences

The *n*th term of an arithmetic sequence in which the first term is $a_1$ and the common difference is *d* is given by the formula:

$$a_n = a_1 + (n-1)d$$

$a_1 =$

$a_2 =$

$a_n =$

| | |
|---|---|
| 1. 3, 7, 11, ….. <br> $a_1 =$ <br> $a_2 =$ <br> $a_4 =$ | 2. Find the $15^{th}$ term in the sequence 3, 7, 11, ….. |

3. Find the $20^{th}$ term of the arithmetic sequence: 3,10,17,24, ...

**Step 1**: Find the common difference.

**Step 2**: Write the equation for the $n^{th}$ term.

**Step 3**: Find the $20^{th}$ term.

4. Write an equation for the $n^{th}$ term of the arithmetic sequence given $a_6 = 11$ and $d = -5$

**Step 1**: Find $a_1$.

**Step 2**: Write the equation for the $n^{th}$ term.

5. Smart Kids has opened a new preschool. It began with 26 students. If they enroll three new students each week, how many students will there be after 15 weeks?

When will they have an enrollment over 100 students?

**Arithmetic means:**

6. Find the arithmetic means in the sequence 21, ____ , ____ , ____ , 45.

7. Find the arithmetic means in the sequence -18, ____ , ____ , ____ , ____ , ____ , 36.

## Part 2: Arithmetic Series

An **arithmetic series** is

The formula for the **sum** of a series is $S_n = n\left(\frac{a_1 + a_n}{2}\right)$

1. Find the sum of 2 + 4 + 6 + …. +100

   **Step 1**: Use the $n^{th}$ term formula to find the number of terms ($n$).

   **Step 2**: Use the sum formula to find the sum of the series.

**Summation notation,** $\sum_{n=1}^{a} rule$

$\sum_{i=1}^{5}(4n-3)$ = ___________ + ___________ + ___________ + ___________ + ___________

**Index:**

**Upper limit of the summation:**

**Lower limit of the summation:**

**Rule:**

**2. Find** $\sum_{k=1}^{14} 3k+2$

**Step 1**: Find the first term of the sequence.

**Step 2**: Find the last term of the sequence.

**Step 3**: Use the summation formula to find the sum.

**3. Find** $\sum_{k=5}^{12} 6k-3$

**Step 1**: Find the first term of the sequence.

**Step 2**: Find the last term of the sequence.

**Step 3**: Use the summation formula to find the sum.

# Section 10-3
# Geometric Sequences and Series

The *n*th term of a geometric sequence in which the first term is $a_1$ and the common ratio is $r$ is given by the formula:

$$a_n = a_1 \cdot r^{n-1}$$

| | |
|---|---|
| 1. 8, 32, 128, …..<br>$a_1$ =<br>$a_2$ =<br>$a_4$ = | 2. Find the 8th term in the sequence 8, 32, 128, ….. |

3. Find $a_n$ of the geometric sequence given $a_1 = -10,\ r = 4,\ n = 12$.

4. Write an equation for the $n^{th}$ term of the geometric sequence: 9, 54, 324, 1944, ….

**Step 1**: Find the common ratio.

**Step 2**: Write the equation for the $n^{th}$ term.

A **geometric series** is

The formula for the **sum** of a series is $S_n = \dfrac{a_1(1-r^n)}{1-r},\ r \neq 1$ when given $a_1$ and *n.*

**5.** Find $\sum_{k=2}^{7} 5(3)^{k-1}$.

**Step 1**: Find the first term of the sequence.

**Step 2:** Identify **r** and **n.**

**Step 3**: Use the summation formula to find the sum.

# Section 10-4
# Infinite Geometric Series

A geometric series that does not end is called an ________________________________ series.

An infinite geometric series has a sum if the common ratio r, is defined by, $-1 < r < 1$.

Convergent:

Divergent:

**Example 1:** Determine whether each geometric infinite series is convergent of divergent.

| | |
|---|---|
| **a.** 2 + 5 + 12.5 + … | **b.** 729 + 243 + 81 + … |

The sum ***S*** of an infinite series with $|r| < 1$ is given by $S = \frac{a_1}{1-r}$.

**Example 2:** Find the sum of each infinite series, if it exists.

| | |
|---|---|
| **a.** 75 + 15 + 3 + … | **b.** $\sum_{k=1}^{\infty} 48\left(\frac{-1}{2}\right)^{k-1}$ |

# Section 10-5
# Recursion and Iteration

In a recursive sequence, each term is ________________________________________.

**Explicit formula:**

**Recursive formula:**

| Recursive Formulas for Sequences |
|---|
| Arithmetic Sequence: |
| Geometric Sequence: |

**Example 1:** Find the first five terms of the sequence in which
$a_1 = 5$ and $a_{n+1} = 2a_n + 7$ , $n \geq 1$ .

**Example 2:** Write a recursive formula for each sequence.

| | |
|---|---|
| a. 3, 10, 17, 24, 31, … | b. 1280, 320, 80, 20, 5, … |
| c. 2, 3, 5, 8, 13, … | d. 4, 9, 19, 39, 79, … |
| e. 2, 3, 6, 18, 108, … | f. 5, 15, 45, 105, 315, … |

**Iteration:**

**Example 3:** Find the first three iterates $x_1$, $x_2$, and $x_3$ of $f(x)=3x-1$ for an initial value of $x_0=5$.

**Example 4:** Find the first three iterates $x_1$, $x_2$, and $x_3$ of $f(x)=-3x+8$ for an initial value of $x_0=6$.